

SUPPLIER PLANNING WITH ANALYTICAL HIERARCHY PROCESS

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Abstract: The aim of this paper is to select the optimal supplier for new equipment according to clients' selected criteria using the analytic hierarchy process (AHP). This paper is composed of a theoretical part, which constitutes a detailed methodology of the AHP an application part, in which the described method is put into practical use for ranking alternatives, the selection of the optimal supplier for a production improvement. The results of the application part are summarized in the conclusion.

1 Introduction

Nowadays is the amount of quality characteristic feature and timely information that we need to know, the optimal response is proportional to the decision we will take and from which the course of the event depends. Every day, in both business and private environments, we encounter difficult situations where decisions need to be made quickly and correctly. Multicriterial decision-making methods are often used to support and simplify decision-making, which provide an efficient apparatus for making the right decisions.

Multiple criteria decision-making (MCDM) has grown as a part of operations research, concerned with designing computational and mathematical tools for supporting the subjective evaluation of performance criteria by decision-makers [1].

Decision making is a process whereby an individual or a group (the decision maker) selects the best alternative from many possible alternatives. It represents an alternative which best meets criteria the decision maker's preferences. Among the important business activities belongs item investment decision making [2].

2 Multi-criteria decision making (MCDM)

Decision Making is the act of choosing between two or more action. Multiple-criteria evaluation problems consist of a finite number of alternatives, explicitly known in the beginning of the solution process. In Multiple criteria design problems (multiple objective mathematical programming problems) the alternatives are not explicitly known. An alternative (solution) can be found by solving a mathematical model. The number of alternatives is either infinite or not countable (when some variables are continuous) or typically very large if countable (when all variables are discrete). But both kind of problems are considered as a subclass of Multi Criteria Decision Making problems. The basic working principle of any MCDM method is same: selection of criteria, selection of alternatives, selection of aggregation methods and

ultimately selection of alternatives based on weights or outranking.

Multicriterial decision-making (evaluation) depends on the choice of the appropriate method. The priority is the method, which results in the decision making based on the quantified usefulness of objects entering the decision-making process. As a result, the decision affects the relevance of the evaluation criteria, and it is therefore necessary to address procedures that allow the weighting of the evaluation criteria to be determined responsibly and accurately.

The problem of multi-criteria evaluation of alternatives is foremost a task involving finding the best (optimal) alternative and ranking the alternatives from the best to the worst conceivable. The fundamental advantages of multi-criteria decision-making methods can be found in the decision maker's ability to evaluate each alternative using many criteria. These methods compel the decision maker to express explicitly (not intuitively) his or her understanding of the importance of each criterion. Therefore, the whole process of the evaluation of alternatives becomes more transparent, easy to follow and clear, for other parties that are more or less engaged in the decision-making process as well [3].

Their common sign is that they estimate multiple options for a possible solution according to different criteria. Based on the nature and method of using the information from the evaluator, it is possible to divide the methods of multi-criteria evaluation into:

- empirical methods,
- heuristic methods,
- exact methods.

Empirical methods are using the knowledge and reality of the decision maker (brainstorming). Heuristic methods are based on subjective evaluation, the results of which are further processed exactly (neural networks). Exact methods are based on scientific analysis (statistical methods).

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Multicriterial methods are widely used in decision-making or evaluation and are addressed by many experts in different fields, so approaches to these methods vary. Among the most widely used approaches to multi-criteria evaluation are those:

- Questionnaire evaluation, in which pairwise comparisons are made.
- Another approach is based on an analytical space calculation where the number of criteria reflects the number of space dimensions.
- Approach based on mathematical-statistical methods.

Several methods of multi-criteria decision-making are known - Decision Matrix Method (DMM), Forced Decision Matrix Method (FDMM), Analytical Hierarchy Process (AHP). The solution first determines the weight of the individual criteria and then quantitatively evaluates how the individual variants of the solution meet the selected criteria. The different methods differ in the method of quantification for both evaluations. The best known decision-making methods include the analytical hierarchical process (AHP) proposed by Thomas L. Saaty [4].

3 Analytic Hierarchy Process

AHP is a method designed to solve complex situations in which the most optimal decision needs to be reached. The basis of this method is divided into the multi-criteria problem, which is divided into smaller parts and then create a hierarchical model. The AHP solves multi-criteria decision-making problems based on a hierarchy. Generally, the hierarchy has three levels: the goal, criteria and alternatives. The criteria can be broken down into sub-criteria to make a lower level. AHP offers a complex and logical concept for problem structuring, the quantification of problem elements that are linked to goals and the evaluation of alternative solutions. It is widespread in several decision-making situations and areas, e.g. industry and commerce. Advantage this method comes from its variability of data evaluation, such as price, supply chain performance, quality, etc. AHP author Thomas L. Saaty was an American mathematician working as a university professor at the University of Pittsburgh [5,6].

The decision making can be structured into three levels:

- the hierarchy,
- the priority,
- the consistency.

The hierarchy

Hierarchy design is a goal definition, identification of alternatives, identification of evaluation factors, assignment of criteria and factor relationships and finishing of the hierarchy. Simultaneously with the creation of a structured hierarchy, an optimized system is developed from a group of criteria (sub-criteria) and alternatives. The

most widely employed illustration of the hierarchy is shown in Figure 1.

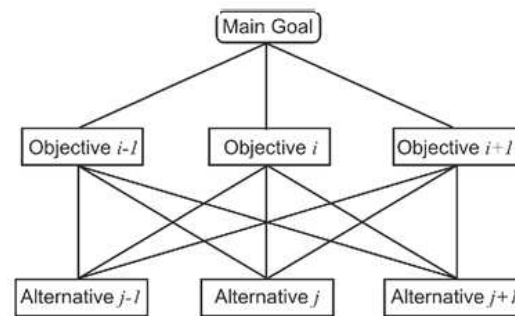


Figure 1 Structure of AHP method (hierarchy)

A hierarchy is a system of classification and organization where each element of the system, except the top one, is subordinate to one or more elements. When creating a structured hierarchy for the AHP method, an optimization system consisting of a main objective, a selected set of factors or criteria and alternatives is set up. This means that we divide the main problem into smaller separate parts. The AHP hierarchy generally has the following levels: main goal, objective (criteria) and alternatives. At the top of the hierarchy is the goal in the middle are the criteria on which we make decisions and below are the alternatives we want to decide on. The breakdown into smaller parts is very important because the evaluation of results by individual sub-criteria is easier, doubts are easier to verify [7].

The priority

Identification of priorities (application of pair-wise comparison, point evaluation of significance, repetition of the procedure for all the hierarchy levels). This step is based on the allocation of points to each paired provocation based on their degree of significance. The paired comparison method is based on the principle of comparing each criterion with each, the preference of the criterion being determined with respect to all other criteria in the set. The identification of priorities (evaluation) is based on expert estimation, in which the factor influences are compared. The scale of evaluation has five basic levels, which are mentioned in Table 1. In this scale, 1 expresses the equally preferred status and 9 expresses the extremely preferred status.

In problem solving, it is very important to assess the criteria preferences [8,9]. The more important is the criterion, the higher is its weight. A few methods exist for criteria normalization (e.g. AHP). Results of the comparison (for each factors pair) were described in term of integer values from 1 (equal value) to 9 (extreme different) where higher number means the chosen factor is considered more important in greater degree than other factor being compared with.

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Table 1 The example scale for comparison

Scale-number of points	Degree of preference	Descriptor
1	Equal importance	Criteria <i>i</i> and <i>j</i> are equal.
3	Moderate importance of one factor over another	Low preference for criterion <i>i</i> before <i>j</i> .
5	Strong or essential importance	Strong preference for criterion <i>i</i> before <i>j</i> .
7	Very strong importance	Very strong preference for criterion <i>i</i> before <i>j</i> .
9	Extreme importance	Absolute preference for criterion <i>i</i> before <i>j</i> .
2, 4, 6, 8	Values for inverse comparison	Medium values between two neighbouring criteria for more precise preference determination.

The information about the significance (s_{ij}) consists of values that determine the ratio of the evaluation criterion's significance in relation to the other criteria. The elements in the matrix s_{ij} are an estimate of the weight ratios of criteria v_i and v_j , so the following applies (1). The pair-wise comparison is conducted between two criteria and the value of preference is noted in a matrix of pair-wise comparisons $S = (s_{ij})$ (2), which has a square shape ($m \times m$). For the elements on the main diagonal of the matrix, the relationship is $s_{ij} = 1$ (each criterion is equal to itself). This matrix is reciprocal, inverse elements are determined by the following formula according to (3)

$$s_{ij} \cong \frac{v_i}{v_j} \tag{1}$$

$$S = \begin{pmatrix} 1 & s_{12} & \dots & s_{1j} \\ \frac{1}{s_{12}} & 1 & \dots & s_{2j} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{s_{1j}} & \frac{1}{s_{2j}} & \dots & 1 \end{pmatrix} \tag{2}$$

$$s_{ij} = \frac{1}{s_{ji}} \tag{3}$$

Weight quantification (weighted values of alternative solutions). In the AHP weights are determined on the basis of (4), under the necessary condition of

$$\sum_{j=1}^m w_j = 1; w_i = \frac{\sqrt[k]{\prod_{j=1}^k s_{ij}}}{\sum_{i=1}^k \sqrt[k]{\prod_{j=1}^k s_{ij}}} \tag{4}$$

The vector w_i is a normalized vector of weights that determines the influence of individual criteria in relation to the parent element.

The consistency

A prerequisite for a correct decision is that the rule of consistency be respected when allocating significance to individual criteria. If this is not the case, it is appropriate for the evaluating body to reconsider its rating. When many pairwise comparisons are performed, some inconsistencies may typically arise. The matrix elements are generally not absolutely consistent. However, the evaluation requires a certain level of matrix consistency, i.e. that the elements are linearly independent. That can be assessed by employing the consistency ratio (CR) as follows (5):

$$CR = \frac{CI}{RI}, CI = \frac{\lambda_{max} - n}{n - 1} \tag{5}$$

CI is the consistency index, λ_{max} is the highest eigenvalue of the matrix and n represents the number of independent rows of the matrix. RI is random index that has different values for a different number of matrix criteria or alternatives, as shown in Table. 2. A consistency ratio lower than 0.1 proves the suitability of the pair-wise comparison matrix.

Table 2 Values of the random index for different numbers of criteria

N	1	2	3	4	5	6	7	8	9	10
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49

The method AHP summary consists of the creation of the hierarchy, weight quantification for each criterion (sub-criterion), comparison of the alternatives according to the identified criteria, analysis of consistency (CR) and determination of the optimal alternative (with the highest aggregate weight). The implementation steps are [10,11]:

- Determination of the problem.
- Determination of the objectives of the problem or consideration of all actors, objectives and its outcome.
- Identification of the criteria for evaluation.

- Structuring the problem in a hierarchy of different levels constituting goal, criteria, sub-criteria and alternatives.
- Comparing each element in the corresponding level and calibrate of them on the 1-9 Saaty scale.
- Performing calculations to find the maximum Eigen value, consistency index (CI), consistency ratio (CR).

The last step is to construct the priority matrix of alternatives and to calculate the overall priority vectors. The overall priority vector of each solution is calculated as

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summation of the priority vector of each alternative multiplication to the respective priority vectors. The alternative with the highest overall priority value provides the result of the analysis.

4 Application

In this study, supplier selection was made for new equipment which is a production improvement. The choice was made among three suppliers. New equipment suppliers are evaluated based on selected criteria:

- C1 - Supplier price,
- C2 - The right quality (equipment),
- C3 - Warranty period provided,
- C4 - Delivery speed, Timeliness of delivery,
- C5 - Reliability of agreements.

a) Pair-wise comparison for all criteria

The priority of each decision alternative with respect to its contribution to different criteria is decided by managers group and is presented in Table 3. In this research, the intensity and importance of each criteria was chosen through a group decision. This sorted out that the supplier price and the right quality has the highest importance intensity, followed by delivery speed, timeliness of delivery while the warranty period provided and reliability of agreements has the lowest importance intensity. The consistency index, the consistency ratio and the priority vector of the synthesized matrix is presented too in Table 3.

Based on references, as presented in Table 2, for a matrix with size of 5, the random consistency ratio RI is 1.12 and the consistency ratio CR is $0.0580 \leq 0.1$. Due to the fact that CR is less than 0.1 the judgments are acceptable. Similarly, all the pair-wise comparison matrices along with the priority vectors for different criteria are calculated as presented in Table 4, Table 5, Table 6, Table 7, Table 8.

Table 3. Pair-wise comparison matrix for five criteria

criteria	C1	C2	C3	C4	C5	$\lambda=5.2599$ CI=0.0650 CR=0.0580≈5.8%	weights	
C1	1	1	4	5	6		K1	0.4771
C2	1	1	3	2	5		K2	0.2793
C3	1/4	1/3	1	1/3	3		K3	0.0901
C4	1/5	1/2	3	1	4		K4	0.1082
C5	1/6	1/5	1/3	1/4	1		K5	0.0452

b) Pair-wise comparison of variants according to criteria

Table 4. Pair-wise comparison matrix for criterion C1

C1	S1	S2	S3	$\lambda=3.0858$ CI=0.0429 RI=0.58 CR=0.0739≈7.3 %	weights	
S1	1	1/3	4		S1	0.2797
S2	3	1	5		S2	0.6267
S3	1/4	1/5	1		S3	0.0936

Table 5. Pair-wise comparison matrix for criterion C2

C2	S1	S2	S3	$\lambda=3.0940$ CI=0.0470 RI=0.58 CR=0.0810≈8.1 %	weights	
S1	1	1/2	1/4		S1	0.1265
S2	2	1	1/5		S2	0.1865
S3	4	5	1		S3	0.6870

Table 6. Pair-wise comparison matrix for criterion C3

C3	S1	S2	S3	$\lambda=3.0649$ CI=0.0324 RI=0.58 CR=0.0559≈5.6 %	weights	
S1	1	3	7		S1	0.6491
S2	1/3	1	5		S2	0.2790
S3	1/7	1/5	1		S3	0.0719

Table 7. Pair-wise comparison matrix for criterion C4

C4	S1	S2	S3	$\lambda=3.1078$ CI=0.0539 RI=0.58 CR=0.0930≈9.3 %	weights	
S1	1	1/4	2		S1	0.2184
S2	4	1	3		S2	0.6301
S3	1/2	1/3	1		S3	0.1515

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Table 8. Pair-wise comparison matrix for criterion 5

C5				$\lambda=3.0536$ CI=0.0268 RI=0.58 CR=0.0462~4.6 %	weights	
S1	1	2	3		S1	0.5278
S2	1/2	1	3		S2	0.3325
S3	1/3	1/3	1		S3	0.1396

Table 9. Priority matrix of alternatives

Criteria	Weight	Supplier		
		S1	S2	S3
C1	0.4771	0.2797	0.6267	0.0936
C2	0.2793	0.1265	0.1865	0.6870
C3	0.0901	0.6491	0.2790	0.0719
C4	0.1082	0.2184	0.6301	0.1515
C5	0.0452	0.5278	0.3325	0.1396
Weigh Sum		0.2748	0.4595	0.2657

Overall priority of the first supplier is 0.2748, of the second supplier is 0.4595 and of third supplier is 0.2657 and this confirms that the second supplier is the preferred solution which can satisfy the criteria (Table 9).

5 Conclusion

The article deals with a detailed analysis of the AHP method - analytical hierarchical process. It describes its origin, popularity, use to date, advantages and disadvantages which this method brings. The procedure for using the AHP method in a simple decision example is also given. The AHP is a very flexible and powerful tool because the scores, and therefore the final ranking, are obtained based on the pairwise relative evaluations of both the criteria and the options provided by the user. The computations made by the AHP are always guided by the decision maker's experience, and the AHP can consequently be considered as a tool that is able to translate the evaluations (both qualitative and quantitative) made by the decision maker into a multicriteria ranking. The AHP method is currently one of the most widely used methods of multi-criteria evaluation, because it is simple, complex, has a wide range of uses, and especially if the decision-maker enters preferential information rationally, this method gives good results. It is only necessary to know how to work with it and how to interpret the obtained data.

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References

- [1] ZAVADSKAS, E.K., TURSKIS, Z., KILDIIENĖ, S.: State of art surveys of overviews on MCDM/MADM methods, *Technological and Economic Development of Economy*, Vol. 20, No. 1, pp. 165-179, 2014. doi:10.3846/20294913.2014.892037
- [2] FOTR, J., SOUČEK, I.: *Investiční rozhodování a řízení projektů*, Praha, Grada Publishing, 2011. (Original in Czech)
- [3] FOTR, J., ŠVECOVÁ, L., et al.: *Manažerské rozhodování postupy, metody a nástroje*, Praha, Ekopress, 2010.
- [4] SAATY, T.L.: *The analytic hierarchy process: Planning, priority setting, resources allocation*, New York, NY: McGraw, 1980.
- [5] Thomas L. Saaty, [Online], Available: http://en.wikipedia.org/wiki/Thomas_L._Saaty [18.11.2019], 2019.
- [6] Analytic hierarchy process, [Online], Available: http://en.wikipedia.org/wiki/Analytic_Hierarchy_Process [19.11.2019], 2019.
- [7] STRAKA, M.: *Distribution and Supply logistics*, Cambridge Scholars Publishing, Newcastle upon Tyne, 2019.
- [8] STRAKA, M.: *Distribučná logistika v príkladoch*, Dekanát - Edičné stredisko/AMS, Fakulta BERG Technickej univerzity v Košiciach, Košice, 2011. (Original in Slovak)
- [9] VARGAS, L.G.: *Models, Methods, Concepts & Applications of the Analytic Hierarchy Process*, 2nd ed., Springer, New York, NY, USA, pp. 1-69, 2012.
- [10] SAATY, T.L.: *Decision making with dependence and feedback: the analytic network process: The organization and prioritization of complexity*, Pittsburgh, Rws Publications, 1996.
- [11] SAATY, T.L.: *Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process*, Pittsburgh, RWS Publications, 2006.

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